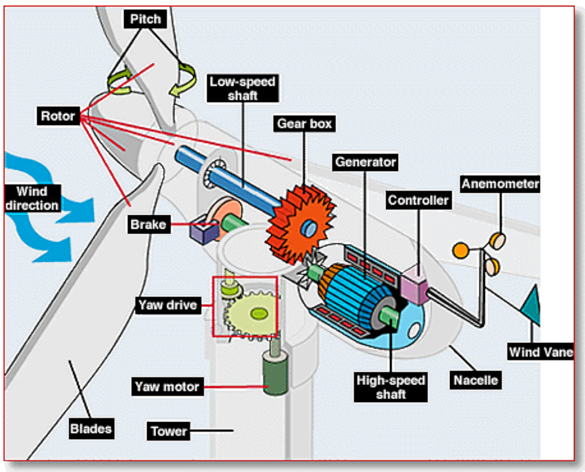
AEM 4321 / EE 4231: HW#3

1. **Wind Turbine Control**

   Wind turbines convert wind energy into electrical energy. The left figure below shows the basic components of a modern wind turbine. Lift is generated on the turbine blades as wind flows past similar to lift on a airplane wing. The lift causes the blades and rotor to rotate. This mechanical rotational energy is converted into electrical energy by a gearbox and generator. The University of Minnesota is currently installing a three-blade, 2.5MW Clipper Liberty turbine (right figure below) at the UMore campus for research purposes.



   The blades on modern wind turbines can be pitched, i.e. turned about their axis as shown in the left figure above. This will increase or decrease the lift forces on the blade depending on the direction of pitch. At high wind speeds, the blades are pitched in order to control the angular velocity of the blades and rotor. A simple model of the turbine dynamics is described by a first-order nonlinear ordinary differential equation of the form:

$$\dot{\omega} = f(\omega, \beta, v) \tag{1}$$

   where ω is the rotor/blade angular velocity (rad/sec), β is the blade pitch angle (degs), and v is the wind speed (m/sec). This model assumes that all three blades are pitched by the same amount. The wind speed v is a disturbance acting on the system. The nonlinear model has the following trim condition: $(\bar{\omega}, \bar{\beta}, \bar{v}) = (3.88$ rad/sec, 16 deg, 18 m/sec). The nonlinear model can be linearized about this trim condition to obtain the following first-order linear model:

$$\dot{x} = -0.28x + 0.05u + 0.07d \tag{2}$$

   where $x(t) := \omega(t) - \bar{\omega}$, $u(t) := \beta(t) - \bar{\beta}$, and $d(t) := v(t) - \bar{v}$.

   Design a proportional controller for this system: $u(t) = K_p(r(t) - x(t))$ where r is the reference angular velocity. Choose K_p so that the closed loop:

   i. is stable,
   ii. has a time constant < 1sec,
   iii. has steady state error $|e_{ss}| < 0.01$ rad/sec when $r(t) = 0.1$ rad/sec and $d(t) = 0$ m/sec for $t \geq 0$,
   iv. has steady state error $|e_{ss}| < 0.05$ rad/sec when $r(t) = 0$ rad/sec and $d(t) = 1$ m/sec for $t \geq 0$,
   v. has $|u(t)| \leq 6$ deg for all $t \geq 0$ when $r(t) = 0.1$ rad/sec and $d(t) = 0$ m/sec for $t \geq 0$.

   Provide justification that your gain K_p satisfies each of the design constraints.

2. **Second Order Response**

   Compute the damping ratio, natural frequency, damped natural frequency and characteristic equation roots for the following eight systems. Then match each of the eight systems with one of the unit step responses in the figure on the next page. All unit step responses in the figure were generated with zero initial conditions. The figure has nine responses so one response will not be matched with any system. You do not need to solve the differential equation to match the system to the response.

i. $\ddot{y} + 11\dot{y} + 10y = u$

ii. $\ddot{y} + 11\dot{y} + 10y = 10u$

iii. $\ddot{y} + 1.2\dot{y} + 1y = 0.3u$

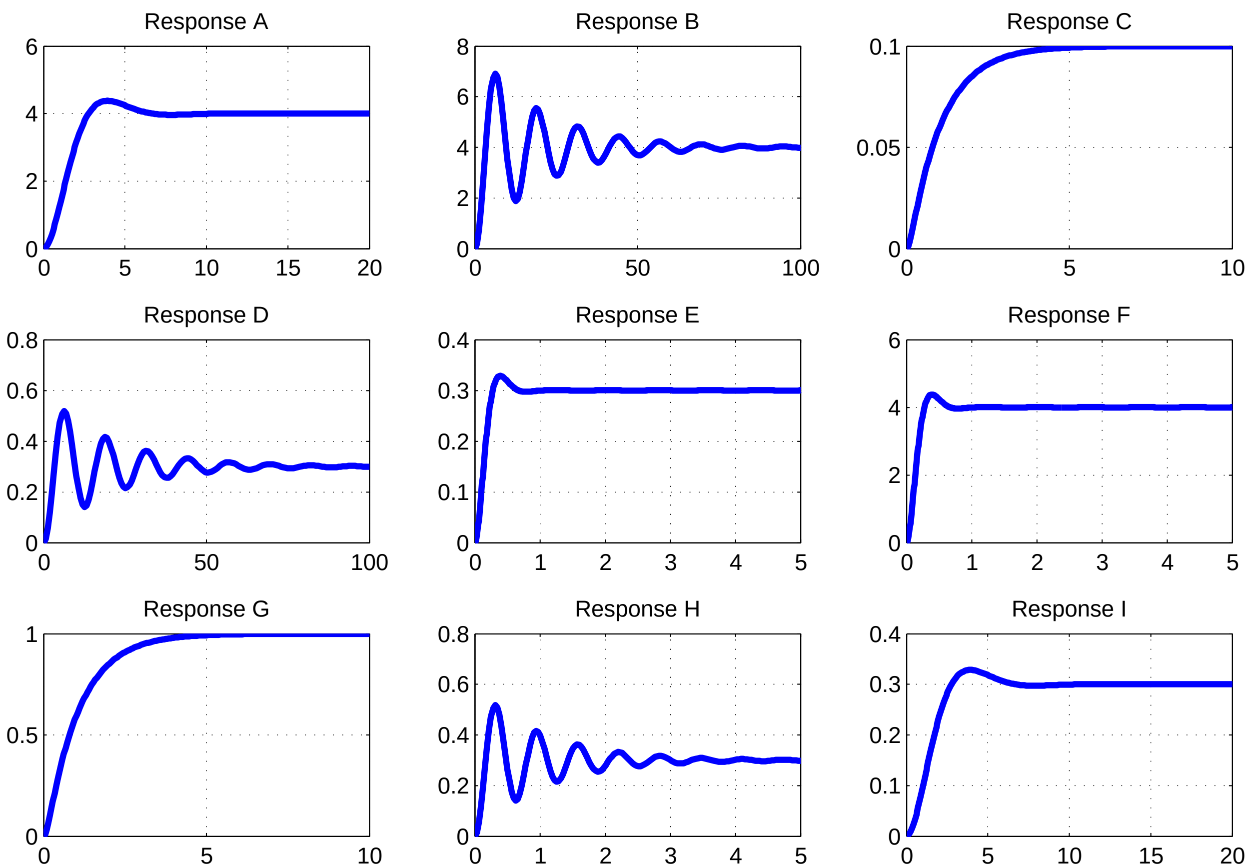
iv. $5\ddot{y} + 6\dot{y} + 5y = 20u$

v. $\ddot{y} + 12\dot{y} + 100y = 30u$

vi. $\ddot{y} + 12\dot{y} + 100y = 400u$

vii. $\ddot{y} + 0.1\dot{y} + 0.25y = 0.075u$

viii. $9\ddot{y} + 0.9\dot{y} + 2.25y = 9u$



3. **PI Control**

Consider the following first order system:

$$\dot{x} = -0.5x + 2u, \qquad x(0) = 0 \tag{3}$$

(a) First, consider a proportional control law $u(t) = K_p(r(t) - x(t))$ where $r(t)$ is the reference command. As mentioned in class, it is typically important, for practical reasons, that $u(t)$ does not get too large. Consider a unit step command:

$$r(t) = \begin{cases} 0 & t < 0 \text{ sec} \\ 1 & t \geq 0 \text{ sec} \end{cases} \tag{4}$$

For what gains K_p is $|u(t)| \leq 1$ for all time?

(b) Choose the gain K_p that satisfies the constraint in part a) and minimizes the steady-state error due to the unit step command. What is the time constant of the closed-loop system for this gain?

(c) Next consider a proportional-integral (PI) control law:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau \tag{5}$$

where $e(t) = r(t) - x(t)$ is the tracking error. Combine the system model (Equation 3) and PI controller (Equation 5) to obtain a model of the closed-loop system in the form:

$$\ddot{x} + a_1 \dot{x} + a_2 x = b_1 \dot{r} + b_2 r \tag{6}$$

How do the damping ratio and natural frequency depend on K_p and K_i? What is the steady state error if r is a unit step?

(d) Keep the value of K_p designed in part b) and choose K_i to obtain a damping ratio of $\zeta = 0.7$. For these PI gains, what are the rise time, maximum overshoot, and 5% settling time?

(e) Construct `Simulink` models for the closed-loop systems with the P and PI controllers. Simulate the closed-loop systems with a unit step r and plot both x vs. t responses on one figure. Do your plots agree with the step response values (time constant, overshoot, etc) computed in the previous parts? If not, then what is the source of the discrepancy? In addition, plot both u vs. t responses on another figure. Does the input for the PI controller still satisfy $|u| \leq 1$ for all time?