AEM 4321 / EE 4231: Exam #2

1. **[30pts]** Figure 1 shows the output response $x(t)$ generated by a linear system $G(s)$ with input $u(t) = A_0 \sin(\omega_0 t)$.

   (a) What are the values of A_0 and ω_0 for the input signal shown in Figure 1?

   (b) What is the magnitude $|G(j\omega_0)|$ in dB?

   (c) What is the phase $\angle G(j\omega_0)$ in degrees?

   The following table converts from actual gain to dB and may be useful:

Actual	100	10	8	5	3	2	1	0.5	0.33	0.2	0.125	0.1	0.01
dB	40	20	18	14	9.5	6	0	-6	-9.5	-14	-18	-20	-40

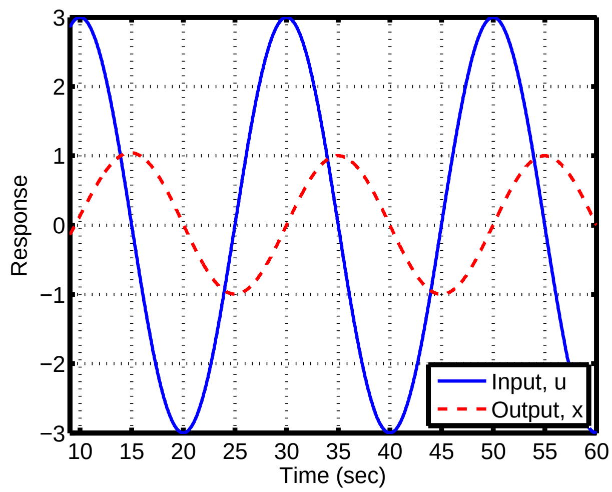

Figure 1: Sinusoidal Input/Output Response

2. **[25pts]** Consider the feedback system in Figure 2 with $G(s) = \frac{4}{s+6}$ and $K(s) = 3$.

   (a) Derive the transfer function from d to e. Express your final answer as a ratio of two polynomials.

   (b) What is the ordinary differential equation that relates input d output e?

   (c) Let $r(t) = 0$ for all $t \geq 0$. Assume the disturbance is $d(t) = A_d \sin(\omega_d t)$ where ω_d is known but A_d is unknown. How can $K(s)$ be modified to achieve $e(t) = 0$ in steady state?

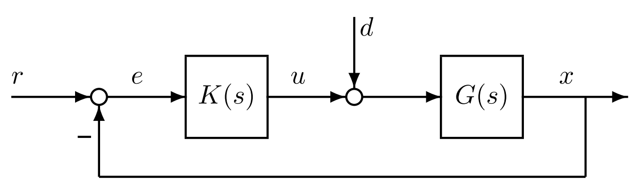

Figure 2: Feedback System

3. **[35pts]** Consider the feedback loop in Figure 3 where $G(s) = \frac{0.1}{s+10}$. The specifications are to design a controller so that: i) the closed-loop is stable, ii) the open loop has a crosssover frequency near 10 rad/sec, and iii) the closed-loop can track $r(t) = \sin(0.01t)$ with less than 1% error.

   (a) Sketch the Bode plot (magnitude and phase) for $G(s)$. Label your plot with the approximate magnitude and phase at the lowest and highest frequencies. Also label the frequencies where your plot changes slope.

   (b) Choose K_p so that $K_pG(s)$ has the desired crossover frequency. You may use your straight-line Bode sketch from Part a) to compute the approximate value of K_p.

   (c) Convert requirement iii) into a requirement on the loop transfer function $L(s) = G(s)K(s)$.

   (d) Does the loop $G(s)K_p$ satisfy the requirement from part (c)? If not, then describe in 1-2 sentences how you would modify your control design to meet this requirement.

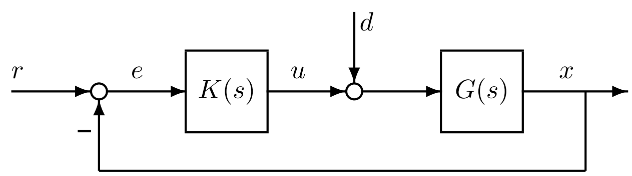

Figure 3: Feedback System

4. **[10pts]** Consider the feedback loop in Figure 3 where $G(s) = \frac{2}{s^2+3s+4}$. Let S and T denote the sensitivity and complementary sensitivity functions of the closed-loop system. The specifications are to design a controller so that: i) the closed-loop is stable, ii) $|S(j\omega)| < 0.1$ for $\omega < 10$rad/sec, and iii) $|T(j\omega)| < 0.1$ for $\omega > 1$rad/sec. Is it possible to achieve these requirements? If so, briefly describe how you would design a controller to meet these requirements.