AEM 4321 / EE 4231: Exam #1

1. Consider the nonlinear system $\dot{x} = f(x, u)$ where $f(x, u) = -2x^3 + xu$
   (a) Find an equilibrium point $(\bar{x}, \bar{u})$ with $\bar{x} = 1$.
   (b) Linearize the system around $(\bar{x}, \bar{u})$. Express your answer as a linear ODE.
   (c) Sketch the approximate solution of $\dot{x} = f(x, u)$ if $u(t) = \bar{u} + 0.4$ for all $t \geq 0$ and $x(0) = \bar{x}$. Label the settling time and steady-state value of x on your sketch.

2. Sketch the response $x(t)$ vs. t for the system, initial conditions, and input given below. Label the steady-state value of x and the approximate settling time. Also label the approximate peak value of x. Specify whether the system is overdamped or underdamped.

$$\ddot{x} + 5\dot{x} + 4x = 8u, \tag{1}$$

$$x(0) = 0, \ \dot{x}(0) = 0, \ u(t) = \begin{cases} 0 & t < 0 \text{ sec} \\ 1 & t \geq 0 \text{ sec} \end{cases} \tag{2}$$

The following table may be useful to sketch the response for an underdamped system.

ζ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$e^{-\pi\zeta/\sqrt{1-\zeta^2}}$	0.729	0.527	0.372	0.254	0.163	0.095	0.046	0.015	0.002

3. Let P denote the system described by $\dot{x} - 3x = u$. Consider the feedback system shown in Figure 1 where K_p and K_f are constant gains. d is a disturbance at the input of P.
   (a) What is the ordinary differential equation (ODE) that models the closed-loop system? Your ODE should have x as the state and include both the the reference r and disturbance d as inputs.
   (b) Choose the gains K_p and K_f so that the closed-loop: i) is stable with time constant $T = 1$sec, and ii) has zero steady state error for a unit step reference ($r(t) = 1$ for $t \geq 0$). You may assume $d = 0$ in this part.
   (c) For the gains selected in part (b), what is the steady-state value of x for a unit step disturbance ($d(t) = 1$ for $t \geq 0$)? You may assume $r = 0$ in this part.
   (d) Is it possible to reduce the effect of the disturbance in part (c) by changing the gain K_f? If so, explain how you would change the gain K_f to reduce the effect of the disturbance.

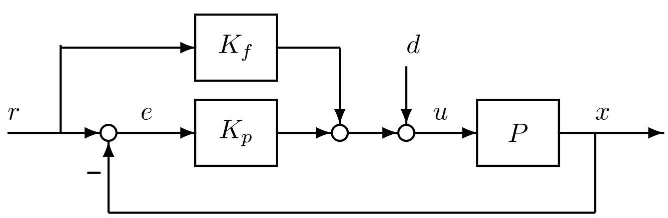

Figure 1: Feedback System Block Diagram

4. Consider the system $2\dot{x} + 6x = 8u$. Let u be given by a PI control law, $u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau$, where $e = r - x$ is the error between the reference and the system output x.
   (a) Derive an ordinary differential equation that models the closed-loop system with input r and output x.
   (b) Choose the gains K_p and K_i so that the closed-loop system is stable and has:
      i. damping ratio equal to $\zeta = 0.5$
      ii. settling time $t_s \leq 3$ sec.
      iii. zero steady state error for a unit step reference.
   (c) Using the gains selected in part (b), what is the peak value of x for a unit step reference? Note: The value will not be an integer or simple fraction but it can be easily computed from the available information.